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More on "La Grande Bouffe": towards higher spin symmetry breaking in AdS

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Abstract: We discuss higher spin gauge symmetry breaking in AdS space from a holographic perspective. Indeed, the AdS/CFT correspondence implies that $\mathcal{N} = 4$ SYM theory at vanishing coupling constant is dual to a theory in AdS which exhibits higher spin gauge symmetry enhancement. When the SYM coupling is non-zero, the current conservation condition becomes anomalous, and correspondingly the local higher spin symmetry in the bulk gets spontaneously broken. In agreement with previous results and holographic expectations, we find that the Goldstone mode responsible for the symmetry breaking in AdS has a non-vanishing mass even in the limit in which the gauge symmetry is restored. Moreover, we show that the mass of the Goldstone mode is exactly the one predicted by the correspondence. Finally, we obtain the precise form of the higher spin supercurrents in the SYM side.

Keywords: Space-Time Symmetries, AdS-CFT and dS-CFT Correspondence, Supersymmetry and Duality.

JHEP08(2005)088

Contents

1. Introduction and summary **1**

2. Spin one: Higgs á la Stückelberg **3**

3. Spin two **4**
- 3.1 s=2 in flat spacetime 4
- 3.2 s=2 in AdS 6

4. Arbitrary Spin **7**
- 4.1 Any s in flat spacetime 7
- 4.2 Any s in AdS 10

5. HS multiplets in superspace **12**
- 5.1 Free $\mathcal{N}=4$ 13
- 5.2 Interactions and anomalies 15

6. Conclusions and perspectives **16**

1. Introduction and summary

The AdS/CFT correspondence [1, 2] between $\mathcal{N}=4$ SYM and type-IIB superstring on $AdS_5 \times S^5$ has received a new wave of attention in the past few years. On the one hand, the BMN limit has suggested the possibility that some 'unprotected' observables on the two sides of the correspondence could be matched in particular regimes [3, 4]. On the other hand, exploiting Higher Spin (HS) symmetry enhancement at small radius [5, 6] and taking the BMN limit as a hint, it is possible to extend the results of [7−9] for the 'first Regge trajectory' to the full string spectrum, showing that it precisely matches with the operator spectrum of free $\mathcal{N}=4$ SYM in the planar limit [10, 11]. When interactions are turned on, i.e. at finite radius but still at vanishing string coupling, all but a handful of higher spin string states become massive by eating lower spin 'Goldstone' fields. This pantagruelic Higgs mechanism, termed *La Grande Bouffe* in [12], is the bulk counterpart of the anomalous violation of classically conserved currents in the boundary CFT. The resolution of the operator mixing yielding the anomalous dimensions [13] has been shown to be equivalent to diagonalizing a dilatation operator [14] that in turn can be identified with the hamiltonian of a supersymmetric spin chain [15]. Integrability of the system at higher loops turns out to be much subtler then expected since it naively leads to inconsistencies with holography [16]. Yet the encouraging results at the Higher Spin enhancement point suggest that Higher Spin Symmetry [7−9, 17−19] could shed some light on this issue and at least help organize *La Grande Bouffe*. In particular decomposing Higher Spin multiplets into superconformal multiplets allows one to combine the latter into long multiplets as expected for unprotected massive states [10, 11].

The aim of this paper is to give a detailed account of the bulk counterpart of this mechanism for totally symmetric spin s fields at the quadratic level in the lagrangian. Although this may sound kinematical, and it is indeed so in so far as mass shifts are left undetermined, we believe Higher Spin Symmetry should constrain if not completely fix the mass shifts. At least it should relate shifts for different spins in the same HS multiplet, very much as group theory fixes the ratios of the masses of the vector bosons after spontaneous breaking of a gauge symmetry. HS currents $J_{i_1 \ldots i_s}$ with $s > 2$ occur in $\mathcal{N} = 4$ SYM where they are conserved and belong to semishort multiplets at vanishing coupling $g = 0$. Interactions are responsible for their anomalous violation

$$\partial^{i_1} J_{i_1 \ldots i_s} = g \mathcal{X}_{i_2 \ldots i_s} \, ,$$

where the anomalous term $\mathcal{X}_{i_2 \ldots i_s}$ lies in a different supermultiplet with respect to $J_{i_1 \ldots i_s}$ at vanishing coupling. Conformal invariance fixes the dimension of such a spin s conserved current on the d dimensional boundary to be $s + d - 2$, when the coupling vanishes. In the same limit the dimension of the anomalous term is $s + d - 1$. This implies that $\mathcal{X}$ is not a conserved spin $s - 1$ current when $g = 0$, and therefore one expects it to be dual to a massive field in the bulk. With this in mind, we therefore study the Higgs mechanism for totally symmetric spin s fields in AdS. Our results are in agreement with the results of [20], where 'partial masslessness' in (A)dS was studied in great detail.[1] We will first describe the Higgs mechanism in flat spacetime, as discussed in [22] and briefly reviewed in [23, 24]. We adopt the Stückelberg description, whereby a massless spin s field eats a massless spin $s - 1$ field, which in turn eats a massless spin $s - 2$ field and so on up to 0 (or 1/2 if the initial field is a fermion). In flat spacetime, when the gauge symmetry is restored, *all* the Goldstone modes become massless, i.e. all the fields from spin s to spin 1 become gauge fields. Holography implies that the same Higgs mechanism works differently in AdS, where the 'Goldstone' spin $s - 1$ field becomes a massive field when the spin s gauge symmetry is restored [21].

The paper is organized as follows. In section 2 we consider spin 1 fields, for which the flat space case and the AdS case actually coincide. We then move in section 3 to spin 2, which for our (holographic) purposes is not to be thought of as the graviton but rather as one of the two extra symmetric tensors that appear in the superstring spectrum at the HS enhancement point (one belongs to the Konishi multiplet and is a superdescendant at level four, while the other is the primary of a spin 2 semishort multiplet). The Higgs mechanism for the graviton in AdS_4 was discussed by Porrati in a very interesting and suggestive series of papers [25]. The crucial point is that in AdS a massive spin 2 field decomposes into a massless spin 2 and a *massive* spin 1 [20, 21]. We then extend the

[1] We thank S. Deser and A. Waldron for pointing out to us their and Zinoviev's [21] contributions to the subject.

JHEP08(2005)088

analysis to the case of arbitrary symmetric tensors in section 4, showing in particular that in AdS a massive spin s field decomposes into a massless spin s and a massive spin $s-1$. Moreover, we show that the mass of the spin $s-1$ Goldstone field is exactly in agreement with the one predicted by the AdS/CFT correspondence. Unfortunately the most general case in AdS for $d \geq 4$ are mixed symmetry tensors that can become 'massless' [26, 27] when they are part of (semi)short multiplets. In order to cope with these, in section 5 we reformulate our kinematical analysis in manifestly supersymmetric terms by carefully identifying the boundary operators and their dual bulk fields participating in the shortening of superconformal multiplets at threshold, i.e. when interactions are turned off and the AdS radius is 'small' and comparable with the string scale. Finally we conclude with remarks and perspectives for future investigation.

2. Spin one: Higgs á la Stückelberg

A very convenient way of describing a massive vector field is through the Stückelberg formulation. One starts with a massless gauge fields A_μ and a massless scalar field χ with an 'axionic' Peccei-Quinn symmetry $\chi \to \chi + \alpha_0$, where α_0 is an arbitrary constant and then 'gauges' the symmetry by means of A_μ. In formulae

$$\delta A_\mu = \partial_\mu \alpha \,, \qquad \delta\chi = m\alpha \,. \tag{2.1}$$

In flat spacetime of any dimension D, a lagrangian invariant under (2.1) is

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2}(\partial_\mu \chi - m A_\mu)(\partial^\mu \chi - m A^\mu) \,, \tag{2.2}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is gauge invariant by construction. Clearly for $m = 0$, χ and A_μ simply decouple and are both massless, while for $m \neq 0$ one can 'gauge χ away' by means of (2.1). As a result the gauge invariant vector field $\mathcal{A}_\mu = A_\mu - \frac{1}{m}\partial_\mu \chi$ is a massive spin one field with the correct number of degrees of freedom $\nu^{m\neq 0}_{s=1} = D-1 = \nu^{m=0}_{s=1} + \nu^{m=0}_{s=0} = D-2+1$.

The nice feature of the above construction is that it admits a straightforward generalization to any curved background without torsion. One simply has to replace partial (∂_μ) with covariant (∇_μ) derivatives and to contract indices with the relevant background metric tensor $g_{\mu\nu}$. The crucial point is that the definition of $F_{\mu\nu}$ is unchanged with respect to its flat spacetime expression.

The lagrangian

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2}(\nabla_\mu \chi - m A_\mu)(\nabla^\mu \chi - m A^\mu) \tag{2.3}$$

is manifestly gauge invariant under

$$\delta A_\mu = \nabla_\mu \alpha \equiv \partial_\mu \alpha \,, \qquad \delta\chi = m\alpha \,. \tag{2.4}$$

To conclude, let us put our findings in a holographic perspective. A massless vector field A_μ in the AdS$_{d+1}$ bulk corresponds to a dimension $\Delta_1 = d-1 = 1+d-2$ classically conserved current J_i in the boundary CFT$_d$, that lives at the unitary bound for spin $s=1$

irreps of SO$(d-2,2)$. A massless scalar field χ in the AdS$_{d+1}$ bulk corresponds to a dimension $\Delta_0 = d = \Delta_1 + 1$ marginal operator $\mathcal{X}$ in the boundary CFT$_d$. The anomalous violation of the current $\partial^i J_i = g\mathcal{X}$ is the boundary counterpart of the Higgs mechanism in the bulk. Two comments are in order. First one can violate a current by a relevant deformation (mass term) of the fixed point boundary action. This is indeed what happens in many interesting RG flows that admit a holographic description in terms of (supersymmetric) domain wall solutions in the bulk [28]. These solutions are only asymptotic to AdS, meaning that (super)conformal invariance is broken explicitly, by deforming the action, or spontaneously, by turning on the VEV's of any (scalar) operator. Here we are interested in HS symmetry breaking patterns compatible with (super)conformal invariance. This is possible only if the free boundary CFT is deformed by an exactly marginal deformation. In $\mathcal{N} = 4$ this corresponds to turning on the (complexified) gauge coupling. All other potentially marginal (single trace[2]) deformations are not integrable, i.e. they acquire anomalous dimensions when they are used to perturb the theory away from the free field theory limit. Second, the masslessness of the Stückelberg field χ is an accident of the spin one case. For spin higher than one in AdS, holography or, more simply, SO$(d-2,2)$ group theory suggest that the relevant spin $s' = s-1$ Stückelberg field be massive. This is precisely what we are going to see in detail in the next two sections.

3. Spin two

3.1 s=2 in flat spacetime

The lagrangian describing the Stückelberg form of a massive spin 2 free field in flat spacetime in D dimensions can be formally obtained starting from a massless spin 2 field in $D+1$ dimensions, described by the lagrangian

$$\mathcal{L} = -\frac{1}{2}(\partial_M \Phi_{NR})^2 + (\partial_N \Phi^N{}_M)^2 + \frac{1}{2}(\partial_M \Phi^L{}_L)^2 + \Phi^L{}_L \partial_M \partial_N \Phi^{MN}\,, \tag{3.1}$$

and allowing the field to depend harmonically on the $D+1$-th coordinate,

$$\Phi(x,y) = \frac{1}{\sqrt{2}}\Phi(x)e^{imy} + \text{c.c.}\,. \tag{3.2}$$

After redefining the D-dimensional fields,

$$\Phi_{\mu\nu} = \phi_{\mu\nu}\,, \qquad \Phi_{\mu y} = -i\chi_\mu\,, \qquad \Phi_{yy} = \psi\,, \tag{3.3}$$

where $\phi_{\mu\nu}$, χ_μ and ψ are chosen to be real, one obtains the lagrangian

$$\begin{aligned}\mathcal{L} = &-\frac{1}{2}(\partial_\mu \phi_{\nu\rho})^2 + (\partial_\nu \phi^\nu{}_\mu)^2 + \frac{1}{2}(\partial_\mu \phi^\lambda{}_\lambda)^2 + \phi^\lambda{}_\lambda \partial_\mu \partial_\nu \phi^{\mu\nu} - \\ &-\frac{m^2}{2}\phi^2_{\mu\nu} + \frac{m^2}{2}(\phi^\lambda{}_\lambda)^2 - (\partial_\mu \chi_\nu)^2 + (\partial_\mu \chi^\mu)^2 +\end{aligned}$$

[2]Exactly marginal double-trace deformations of the Leigh-Strassler type [29] are known to exist [30] and their effects have been studied in [31].

JHEP08(2005)088

$$+\partial_\mu\psi\partial^\mu\phi^\lambda{}_\lambda+\psi\partial_\mu\partial_\nu\phi^{\mu\nu}+2m\chi_\mu\partial_\nu\phi^{\mu\nu}+2m\phi^\lambda{}_\lambda\partial_\mu\chi^\mu\,, \tag{3.4}$$

that is invariant with respect to the gauge transformations

$$\delta\phi_{\mu\nu}=2\partial_{(\mu}\epsilon_{\nu)}\,,\qquad \delta\chi_\mu=\partial_\mu\alpha-m\epsilon_\mu\,,\qquad \delta\psi=2m\alpha\,. \tag{3.5}$$

If m is non-vanishing, one can use both ϵ_μ and α to gauge away χ_μ and ψ, so that the resulting lagrangian describes a massive spin 2 field. If m vanishes instead, the lagrangian describes massless spin 2, spin 1 and spin 0 fields.[3]

We will see in the next subsection which changes are needed when one considers AdS instead of flat spacetime. In this direction, it is very instructive to prove explicitly gauge invariance of the lagrangian (3.4) with respect to the transformations (3.5), and it is actually easier to consider the field equations

$$\begin{aligned}\Box\phi_{\mu\nu}-2\partial_{(\mu}(\partial\cdot\phi)_{\nu)}+\partial_\mu\partial_\nu\phi^\lambda{}_\lambda-\eta_{\mu\nu}[\Box\phi^\lambda{}_\lambda-\partial\cdot\partial\cdot\phi]-&\\ -m^2\phi_{\mu\nu}+m^2\eta_{\mu\nu}\phi^\lambda{}_\lambda-2m\partial_{(\mu}\chi_{\nu)}+2m\eta_{\mu\nu}(\partial\cdot\chi)-\eta_{\mu\nu}\Box\psi+\partial_\mu\partial_\nu\psi&=0\,, \qquad (3.6)\\ \Box\chi_\mu-\partial_\mu(\partial\cdot\chi)+m(\partial\cdot\phi)_\mu-m\partial_\mu\phi^\lambda{}_\lambda&=0\,, \qquad (3.7)\\ \Box\phi^\lambda{}_\lambda-\partial_\mu\partial_\nu\phi^{\mu\nu}&=0\,, \qquad (3.8)\end{aligned}$$

obtained varying (3.4) with respect to $\phi_{\mu\nu}$, χ_μ and ψ respectively, instead of the lagrangian itself.

If $m\neq 0$, one is allowed to perform the redefinition of χ_μ

$$\chi'_\mu=\chi_\mu-\frac{1}{2m}\partial_\mu\psi\,, \tag{3.9}$$

such that the gauge transformation of the new field does not contain α anymore:

$$\delta\chi'_\mu=-m\epsilon_\mu\,. \tag{3.10}$$

In terms of the new field (we will drop the 'prime' index from now on), the equations are

$$\begin{aligned}\Box\phi_{\mu\nu}-2\partial_{(\mu}(\partial\cdot\phi)_{\nu)}+\partial_\mu\partial_\nu\phi^\lambda{}_\lambda-\eta_{\mu\nu}[\Box\phi^\lambda{}_\lambda-\partial\cdot\partial\cdot\phi]-&\\ -m^2\phi_{\mu\nu}+m^2\phi^\lambda{}_\lambda-2m\partial_{(\mu}\chi_{\nu)}+2m\eta_{\mu\nu}(\partial\cdot\chi)&=0\,, \qquad (3.11)\\ \Box\chi_\mu-\partial_\mu(\partial\cdot\chi)+m(\partial\cdot\phi)_\mu-m\partial_\mu\phi^\lambda{}_\lambda&=0\,, \qquad (3.12)\\ \Box\phi^\lambda{}_\lambda-\partial_\mu\partial_\nu\phi^{\mu\nu}&=0\,. \qquad (3.13)\end{aligned}$$

There is no field varying with respect to α, while the gauge invariance with respect to ϵ_μ is straightforward. This field redefinition is nothing but a gauge transformation that removes ψ. An analogous gauge transformation can be used to remove χ_μ, so that once all gauge invariances are fixed one is left with a massive spin 2 field. The reason why we perform explicitly this field redefinition for χ_μ only (or for the field of rank $s-1$ only when we consider the spin s case in the next section) is related to our expectation that χ_μ become massive in the AdS case, as we are going to show in the next subsection.

[3] A redefinition of $\phi_{\mu\nu}$ of the form $\phi_{\mu\nu}=\phi'_{\mu\nu}-\frac{1}{D-2}\eta_{\mu\nu}\psi$ is needed in order to obtain the standard kinetic term for the scalar, so that in the lagrangian (3.4) the fields decouple for $m=0$.

If $m = 0$, the gauge parameters can not be used to gauge away ψ and χ_μ anymore. Correspondingly, there is an inconsistency in the redefinition of χ_μ in eq. (3.9), which appears in the form of gauge symmetry enhancement,

$$\delta\chi_\mu = \partial_\mu \alpha' \tag{3.14}$$

in eq. (3.12). As we will see, this is the main difference with respect to the AdS case: if the curvature is non-vanishing, a $1/L$ mass term for χ_μ appears,[4] so that the field redefinition removing the gauge variation with respect to α continues to be consistent at $m = 0$, since there is no gauge symmetry enhancement.

3.2 s=2 in AdS

Now we move on to the AdS case. We want to modify the equations (3.11), (3.12) and (3.13) in such a way that they will turn out to be invariant with respect to the gauge transformations

$$\delta\phi_{\mu\nu} = 2\nabla_{(\mu}\epsilon_{\nu)}\,, \qquad \delta\phi_\mu = -m\epsilon_\mu\,. \tag{3.15}$$

The equations will be modified because the commutator of two covariant derivatives in AdS is

$$[\nabla_\mu, \nabla_\nu]V_\rho = -R^\lambda{}_{\rho\mu\nu}V_\lambda = \frac{1}{L^2}(g_{\nu\rho}V_\mu - g_{\mu\rho}V_\nu)\,. \tag{3.16}$$

Let us consider eqs. (3.11) and (3.13) first. The variation of the third line of eq. (3.11) does not involve any commutator, while the first two lines, as well as the whole of eq. (3.13), become gauge invariant adding a $1/L^2$ AdS mass term for $\phi_{\mu\nu}$. The result is

$$\begin{aligned} &\Box\phi_{\mu\nu} - 2\nabla_{(\mu}(\nabla\cdot\phi)_{\nu)} + \nabla_{(\mu}\nabla_{\nu)}\phi^\lambda{}_\lambda + \frac{2}{L^2}(\phi_{\mu\nu} - g_{\mu\nu}\phi^\lambda{}_\lambda) - \\ &-m^2\phi_{\mu\nu} + m^2 g_{\mu\nu}\phi^\lambda{}_\lambda - 2m\nabla_{(\mu}\chi_{\nu)} + 2m g_{\mu\nu}(\nabla\cdot\chi) - \\ &-g_{\mu\nu}\left[\Box\phi^\lambda{}_\lambda - \nabla\cdot\nabla\cdot\phi - \frac{D-1}{L^2}\phi^\lambda{}_\lambda\right] = 0\,, \end{aligned} \tag{3.17}$$

$$\Box\phi^\lambda{}_\lambda - \nabla_{(\mu}\nabla_{\nu)}\phi^{\mu\nu} - \frac{D-1}{L^2}\phi^\lambda{}_\lambda = 0\,. \tag{3.18}$$

Now consider eq. (3.12). Observe first of all that if we write the kinetic term (first two terms) in the form

$$\nabla_\nu F^\nu{}_\mu = \Box\chi_\mu - \nabla_\nu\nabla_\mu\chi^\nu\,, \tag{3.19}$$

where we have commuted the indices μ and ν in the derivatives, this term is gauge invariant with respect to $\delta\chi_\mu = \nabla_\mu\alpha$ without the addition of any AdS mass term. In other words, if we start with this equation and we perform a field redefinition of χ_μ like in eq. (3.9), we are not going to produce terms containing ψ. This means that we just have to consider the expression

$$\Box\chi_\mu - \nabla_\nu\nabla_\mu\chi^\nu + m(\nabla\cdot\phi)_\mu - m\nabla_\mu\phi^\lambda{}_\lambda\,, \tag{3.20}$$

and see whether it is invariant with respect to the gauge parameter ϵ_μ or not.

[4] This is a proper mass term, in the sense that it breaks gauge invariance. Recall that in AdS gauge invariant field equations require a mass-like term, which we refer to as the AdS mass in the following.

The variation of eq. (3.20) is

$$-2m[\nabla_\mu, \nabla_\nu]\epsilon^\nu = -\frac{2m}{L^2}(D-1)\epsilon_\mu\,, \tag{3.21}$$

so that the gauge invariant field equation is

$$\Box\chi_\mu - \nabla_\nu\nabla_\mu\chi^\nu + m(\nabla\cdot\phi)_\mu - m\nabla_\mu\phi^\lambda{}_\lambda - \frac{2(D-1)}{L^2}\chi_\mu = 0\,. \tag{3.22}$$

We therefore obtain a mass term for χ_μ. Nothing changes if $m \neq 0$, since we can gauge away χ_μ and end up with a massive spin 2 field. When $m = 0$ instead, the spin 2 field becomes massless, as one can see from eq. (3.17), while eq. (3.22) still describes a massive spin 1 field when $m = 0$, and there is no gauge symmetry enhancement, unlike the flat spacetime case. We therefore end up with a massless spin 2 and with a massive spin 1 [20, 21].

The same technique can be applied to any spin s (symmetric tensor with s spacetime indices), to show that in AdS a massive spin s field decomposes into a massless spin s and a massive spin $s-1$ in the limit of vanishing spin s mass. This is what we are going to show in the next section.

4. Arbitrary Spin

4.1 Any s in flat spacetime

In order to derive the Stückelberg formulation of a massive spin s field in D dimensions, we proceed in a way similar to the spin 2 case of the previous section. We consider a massless spin s field in $D+1$ dimensions, described in terms of a symmetric rank s tensor $\Phi_{M_1\ldots M_s}$ (M_i are $D+1$-dimensional spacetime indices) satisfying the condition

$$\Phi^L{}_L{}^M{}_{M\ldots} = 0\,, \tag{4.1}$$

and whose lagrangian [32]

$$\begin{aligned}\mathcal{L} = &-\frac{1}{2}(\partial_M\Phi^{(s)}_{\ldots})^2 + \frac{s}{2}(\partial\cdot\Phi^{(s)}_{\ldots})^2 + \frac{s(s-1)}{4}(\partial_M\Phi^{(s)L}{}_{L\ldots})^2 + \\ &+\frac{s(s-1)(s-2)}{8}(\partial\cdot\Phi^{(s)L}{}_{L\ldots})^2 + \frac{s(s-1)}{2}\Phi^{(s)L}{}_{L\ldots}(\partial\cdot\partial\cdot\Phi^{(s)}_{\ldots})\end{aligned} \tag{4.2}$$

is invariant with respect to the gauge transformations[5]

$$\delta\Phi_{M_1\ldots M_s} = s\partial_{(M_1}\epsilon_{M_2\ldots M_s)}\,, \tag{4.3}$$

where ϵ is symmetric traceless with $(s-1)$ indices. As we did in the previous section, we want to obtain the equations for a massive field in D dimension in the Stückelberg formulation by means of a KK dimensional reduction. We therefore consider the field to depend harmonically on the $D+1$-th coordinate,

$$\Phi_{\mu_1\ldots\mu_{s-k}y\ldots y}(x,y) = (i)^k\phi^{(s-k)}_{\mu_1\ldots\mu_{s-k}}(x)e^{imy} + \text{c.c.}\,, \tag{4.4}$$

[5] Parentheses (...) denote symmetrization of spacetime indices with strength one.

and by taking linear combinations we can choose the fields $\phi^{(s-k)}$ to be real in D dimensions. It is convenient to perform this KK reduction directly on the equation of motion, unlike the previous section, where the reduction was performed on the lagrangian.[6]

We thus consider the equation

$$\Box\Phi_{M_1\ldots M_s} - s\partial_{(M_1}(\partial\cdot\Phi)_{M_2\ldots M_s)} + \frac{s(s-1)}{2}\partial_{(M_1}\partial_{M_2}\Phi^L{}_{LM_3\ldots M_s)} = 0\,, \tag{4.5}$$

describing a massless spin s field in $D+1$ dimensions, obtained varying the lagrangian of eq. (4.2). We want to consider the dimensional reduction of this equation to D dimensions. One gets

$$\begin{aligned}&[\Box - \frac{(k-1)(k-2)}{2}m^2]\phi^{(s-k)}_{\mu_1\ldots\mu_{s-k}} - (s-k)\partial_{(\mu_1}(\partial\cdot\phi^{(s-k)})_{\mu_2\ldots\mu_{s-k})} -\\ &-(k-1)(s-k)m\partial_{(\mu_1}\phi^{(s-k-1)}_{\mu_2\ldots\mu_{s-k})} - km(\partial\cdot\phi^{(s-k+1)})_{\mu_1\ldots\mu_{s-k}} +\\ &+\frac{(s-k)(s-k-1)}{2}\partial_{(\mu_1}\partial_{\mu_2}\phi^{(s-k)\lambda}{}_{\lambda\mu_3\ldots\mu_{s-k})} - \frac{(s-k)(s-k-1)}{2}\partial_{(\mu_1}\partial_{\mu_2}\phi^{(s-k-2)}_{\mu_3\ldots\mu_{s-k})} +\\ &+k(s-k)m\partial_{(\mu_1}\phi^{(s-k+1)\lambda}{}_{\lambda\mu_2\ldots\mu_{s-k})} + \frac{k(k-1)}{2}m^2\phi^{(s-k+2)\lambda}{}_{\lambda\mu_1\ldots\mu_{s-k}} = 0\,,\end{aligned} \tag{4.6}$$

where $k = 0, 1, \ldots, s$, and from the $D+1$-dimensional gauge transformation of eq. (4.3) after having defined the D dimensional gauge parameters

$$\epsilon_{\mu_1\ldots\mu_{s-k-1}y\ldots y} = (i)^k\epsilon^{(s-k-1)}_{\mu_1\ldots\mu_{s-k-1}}\,, \tag{4.7}$$

one obtains the gauge transformations of the D-dimensional fields $\phi^{(s-k)}$,

$$\delta\phi^{(s-k)}_{\mu_1\ldots\mu_{s-k}} = (s-k)\partial_{(\mu_1}\epsilon^{(s-k-1)}_{\mu_2\ldots\mu_{s-k})} + km\epsilon^{(s-k)}_{\mu_1\ldots\mu_{s-k}}\,. \tag{4.8}$$

The traceless condition for ϵ becomes

$$\epsilon^{(s-k)\lambda}{}_{\lambda\ldots} - \epsilon^{(s-k-2)}_{\ldots} = 0\,, \tag{4.9}$$

while the constraint (4.1) becomes

$$\phi^{(s-k)\lambda}{}_{\lambda}{}^{\rho}{}_{\rho\ldots} - 2\phi^{(s-k-2)\lambda}{}_{\lambda\ldots} + \phi^{(s-k-4)}{}_{\ldots} = 0 \qquad (k = 0, \ldots, s-4)\,. \tag{4.10}$$

If $m \neq 0$, some of the lower spin fields can be put to zero fixing their gauge invariance, while some others can not be gauged away because of the constraint (4.9) on the gauge parameters. These fields, that are identically zero on shell, are the auxiliary fields of the massive theory, and one ends up with an equation for a massive spin s field $\phi^{(s)}$.

[6] This is the reason why the equations we obtain in this section for $s = 2$ are apparently different to the equations of the previous section. One can check that they are equivalent for instance by adding eq. (3.8) to the trace of eq. (3.6).

The occurrence of auxiliary fields in lagrangians for massive spin s fields can be understood considering the example of a massive spin 2 field, whose field equation is

$$(\Box - m^2)\Phi_{\mu\nu} = 0\,, \qquad (\partial \cdot \Phi)_\mu = 0\,, \tag{4.11}$$

where $\Phi_{\mu\nu}$ is symmetric and traceless. These equations can be obtained from the lagrangian

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu \Phi_{\nu\rho})^2 + (\partial \cdot \Phi)^2_\mu - \frac{m^2}{2}\Phi^2_{\mu\nu} \tag{4.12}$$

only if the constraint $(\partial \cdot \partial \cdot \Phi) = 0$ is imposed. It is therefore necessary to include a Lagrange multiplier, that is an auxiliary field Φ, whose field equation gives $\Phi = 0$ and $(\partial \cdot \partial \cdot \Phi) = 0$.[7] One therefore considers the lagrangian

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu \Phi_{\nu\rho})^2 - \frac{m^2}{2}\Phi^2_{\mu\nu} + (\partial \cdot \Phi)^2_\mu - \frac{2}{3}\Big[-\frac{1}{2}((\partial_\mu \Phi)^2 + 2m^2\Phi^2) + \Phi_{\mu\nu}\partial^\mu\partial^\nu\Phi\Big]\,, \tag{4.13}$$

giving rise to a system of two equations for Φ and $(\partial \cdot \partial \cdot \Phi) = 0$, whose determinant

$$\det \begin{pmatrix} 2(\frac{1}{2}\Box - m^2) & 1 \\ -\frac{1}{2}\Box^2 & (-m^2 - \frac{1}{2}\Box) \end{pmatrix} = 2m^4 \tag{4.14}$$

is *algebraic* and non-vanishing, and thus admits $\Phi = 0$ and $(\partial \cdot \partial \cdot \Phi) = 0$ as the only solution.

In [33] it was shown how this can be generalized to any (integer and half-integer) spin s field. In the case of integer spin the theory is described in terms of a field $\Phi^{(s)}$, symmetric and traceless, and a set of auxiliary fields $\Phi^{(s-k)}$, with $k = 2, 3, \ldots, s$, again symmetric and traceless. This is precisely consistent with what we get in the Stückelberg formulation [22], where the auxiliary fields are the components of the lower rank fields that can not be put to zero fixing the gauge because of the constraint (4.9). It is interesting to consider in detail the spin 3 case, which is the first for which the constraint (4.9) is non-trivial. After gauging away $\phi^{(0)}$, $\phi^{(1)}_\mu$ and the traceless part of $\phi^{(2)}_{\mu\nu}$, one is left with the trace of $\phi^{(2)}$ and with the spin 3 field $\phi^{(3)}_{\mu\nu\rho}$. After redefining the fields, this is the same as a traceless rank 3 field, together with a rank 1 field (the trace of $\phi^{(3)}$) and a rank 0 field, which are precisely the auxiliary fields of [33]. The procedure of showing that the Stückelberg formulation and the one of ref. [33] are equivalent was explicitly carried out for the spin 3 case in ref. [22].

Coming back to our equations for arbitrary s, if $m = 0$ the situation is the same as the one we encountered for spin 2, since none of the gauge parameters can be used to gauge away any of the fields, and therefore all the fields $\phi^{(s-k)}$, $k = 0, 1, \ldots, s$, become massless. In order to see this from our equations, one has to perform recursive field redefinitions, so that eq. (4.9) becomes a traceless condition for the redefined parameters, while eq. (4.10) becomes a double-traceless condition for the redefined fields, whose gauge transformations look exactly like eqs. (4.8) with $m = 0$ in terms of the new parameters.

[7] In the notation of the previous section, this auxiliary field was the trace of the rank-2 field itself.

4.2 Any s in AdS

We now want to consider the same system of equations in AdS. Generalizing the spin 2 case, we are confident that all the fields can be gauged away up to spin $s-1$, even when $m=0$.[8] We do not worry for the moment about the constraints on the gauge parameters (4.7) and on the fields (4.1), we will comment on them at the end of this subsection.

The idea is the following: we consider the field equation for $\phi^{(s-1)}$, and we gauge away $\phi^{(s-2)}$ and $\phi^{(s-3)}$ using $\epsilon^{(s-2)}$ and $\epsilon^{(s-3)}$. This implies that only the fields $\phi^{(s)}$ and $\phi^{(s-1)}$ will appear in the equation, while all the other fields $\phi^{(s-k)}$, with $k=4,\ldots,s$ are auxiliary fields. The only gauge invariance left is the one with respect to the *traceless* gauge parameter $\epsilon^{(s-1)}$, and in AdS it will require the addition of a mass term for $\phi^{(s-1)}$. We then go to the $m=0$ limit, and we see whether the mass term we included is the AdS mass or not. If the mass term is equal to the AdS mass, this means that we have a gauge symmetry enhancement and our procedure is inconsistent. If the mass term is different from the AdS mass, this instead means that we can continue the procedure to $m=0$, and we end up with a massless spin s and a massive spin $s-1$.

Before we proceed, we first derive the AdS mass for a spin s field [17, 34]. We consider the variation of the equation

$$\Box\phi^{(s)}_{\mu_1\ldots\mu_s} - s\nabla_{(\mu_1}(\nabla\cdot\phi^{(s)})_{\mu_2\ldots\mu_s)} + \frac{s(s-1)}{2}\nabla_{(\mu_1}\nabla_{\mu_2}\phi^{(s)\lambda}{}_{\lambda\mu_3\ldots\mu_s)} - \\ -M^2_{AdS}\,\phi^{(s)}_{\mu_1\ldots\mu_s} - \tilde{M}^2_{AdS} g_{(\mu_1\mu_2}\phi^{(s)\lambda}{}_{\lambda\mu_3\ldots\mu_s)} = 0 \qquad (4.15)$$

with respect to

$$\delta\phi^{(s)}_{\mu_1\ldots\mu_s} = s\nabla_{(\mu_1}\epsilon^{(s-1)}_{\mu_2\ldots\mu_s)}\,, \qquad (4.16)$$

where ϵ is traceless. The gauge variation of the derivative part of the equation is

$$s[\Box,\nabla_{(\mu_1}]\epsilon_{\mu_2\ldots\mu_s)} + s(s-1)\nabla_{(\mu_1}[\nabla_{\mu_2},\nabla_\rho]\epsilon^\rho{}_{\mu_3\ldots\mu_s)}\,. \qquad (4.17)$$

Substituting the AdS Riemann tensor, we get

$$\frac{s(s-2)(D-1)+s(s-1)(s-4)}{L^2}\nabla_{(\mu_1}\epsilon_{\mu_2\ldots\mu_s)} + \frac{2s(s-1)}{L^2}g_{(\mu_1\mu_2}(\nabla\cdot\epsilon)_{\mu_3\ldots\mu_s)}\,. \qquad (4.18)$$

This contribution is cancelled by the variation of the AdS mass term in (4.15) if

$$M^2_{AdS} = \frac{(s-2)(D-1)+(s-1)(s-4)}{L^2}\,, \qquad \tilde{M}^2_{AdS} = \frac{s(s-1)}{L^2}\,. \qquad (4.19)$$

Since the trace of the field is zero on shell choosing a suitable gauge [34], the relevant AdS mass term is M^2_{AdS}.

We now come back to our problem, and consider the equation for the spin $s-1$ Stückelberg field $\phi^{(s-1)}$ of the previous subsection, that we denote here by $\chi^{(s-1)}$ in order to adhere with the notation in sections 1 and 2. The expression that we get by substituting

[8] See [20] and [21] for similar results.

derivatives with covariant derivatives in (4.6) with $k = 1$ and by removing all the lower rank fields is

$$\begin{aligned}
&\Box\chi^{(s-1)}_{\mu_1\dots\mu_{s-1}} - (s-1)\nabla_{(\mu_1}(\nabla\cdot\chi^{(s-1)})_{\mu_2\dots\mu_{s-1})} + \\
&+\frac{(s-1)(s-2)}{2}\nabla_{(\mu_1}\nabla_{\mu_2}\chi^{(s-1)\lambda}{}_{\lambda\mu_3\dots\mu_{s-1})} - \\
&-m(\nabla\cdot\phi^{(s)})_{\mu_1\dots\mu_{s-1}} + (s-1)m\nabla_{(\mu_1}\phi^{(s)\lambda}{}_{\lambda\mu_2\dots\mu_{s-1})}\,,
\end{aligned} \tag{4.20}$$

whose variation with respect to

$$\delta\phi^{(s)}_{\mu_1\dots\mu_s} = s\nabla_{(\mu_1}\epsilon_{\mu_2\dots\mu_s)}\,, \qquad \delta\chi^{(s-1)}_{\mu_1\dots\mu_{s-1}} = m\epsilon_{\mu_1\dots\mu_{s-1}} \tag{4.21}$$

is

$$m(s-1)[\nabla_{(\mu_1},\nabla_\rho]\epsilon^{\rho}{}_{\mu_2\dots\mu_{s-1})} = \frac{m(s-1)[(D-1)+(s-2)]}{L^2}\epsilon_{\mu_1\dots\mu_{s-1}}\,, \tag{4.22}$$

assuming that ϵ is traceless. The mass term that cancels this variation is

$$-\frac{(s-1)[(D-1)+(s-2)]}{L^2}\chi^{(s-1)}_{\mu_1\dots\mu_{s-1}}\,, \tag{4.23}$$

so that one ends up with the equation

$$\begin{aligned}
&\Box\chi^{(s-1)}_{\mu_1\dots\mu_{s-1}} - (s-1)\nabla_{(\mu_1}(\nabla\cdot\chi^{(s-1)})_{\mu_2\dots\mu_{s-1})} + \\
&+\frac{(s-1)(s-2)}{2}\nabla_{(\mu_1}\nabla_{\mu_2}\chi^{(s-1)\lambda}{}_{\lambda\mu_3\dots\mu_{s-1})} - \\
&-m(\nabla\cdot\phi^{(s)})_{\mu_1\dots\mu_{s-1}} + (s-1)m\nabla_{(\mu_1}\phi^{(s)\lambda}{}_{\lambda\mu_2\dots\mu_{s-1})} - \\
&-\frac{(s-1)[(D-1)+(s-2)]}{L^2}\chi^{(s-1)}_{\mu_1\dots\mu_{s-1}} = 0\,.
\end{aligned} \tag{4.24}$$

We thus would like to compare the mass term that we obtained to the AdS mass term, that is the first of eqs. (4.19), where s has to be substituted with $s-1$. They are definitely different, which means that no symmetry enhancement occurs when $m = 0$, and any massive spin s field in the limit of zero mass decomposes into a massless spin s field and a massive spin $s-1$ field. In other words, the new feature of AdS is the fact that the auxiliary field structure is preserved for the spin $s-1$ field even when the spin s field becomes massless. Note that our procedure leaves undetermined a possible mass term of the form $1/L^2 g_{(\mu_1\mu_2}\chi^{(s-1)\lambda}{}_{\lambda\mu_3\dots\mu_s)}$ in eq. (4.24), since $\chi^{(s-1)\lambda}{}_\lambda$ is gauge invariant with respect to (4.21). This is not an issue as long as one is focused on the field equations, since we expect $\chi^{(s-1)\lambda}{}_\lambda$ to be zero on shell using the lower rank equations. Nevertheless, the whole set of equations should be derivable from a lagrangian once the correct equations for the auxiliary fields are introduced, in a similar way to the flat space case.

The difference between the AdS mass term and this mass term (for simplicity we define $s' = s-1$ from now on) is

$$-\frac{2(D-1)+4(s'-1)}{L^2}\chi^{(s')}_{(\mu_1\dots\mu'_s)}\,. \tag{4.25}$$

We therefore get

$$M^2L^2 = 2(D-1)+4(s'-1)\,. \tag{4.26}$$

In $D = 5$ $(d = 4)$ this equation becomes

$$M^2L^2 = 4(s' + 1)\,. \tag{4.27}$$

This is exactly what we get from the standard relation between mass in AdS and dimension of the dual operator in the boundary theory [35, 12],

$$M^2L^2 = \Delta(\Delta - 4) - \Delta_{\min}(\Delta_{\min} - 4)\,, \tag{4.28}$$

with

$$\Delta = s' + 4\,, \qquad \Delta_{\min} = s' + 2\,, \tag{4.29}$$

which is exactly the dimension of the corresponding spin s' operator at vanishing Yang-Mills coupling. For arbitrary dimension $d = D-1$, $\Delta_{\min} = s' + d - 2$ represents the unitary bound for the dimension of the HWS (actually lowest weight state, but we are physicists) of the UIR with spin s', i.e. a totally symmetric rank s' classically conserved tensor current, and the identity (4.28), with 4 substituted with d, is satisfied with $\Delta = s' + d$.

5. HS multiplets in superspace

Higher spin (HS) currents $J_{i_1 \ldots i_s}$ and their anomalous terms $\mathcal{X}_{i_2 \ldots i_s}$

$$\partial^{i_1} J_{i_1 \ldots i_s} = g\mathcal{X}_{i_2 \ldots i_s} \tag{5.1}$$

occur in $\mathcal{N} = 4$ SYM where, at vanishing coupling $g = 0$, the HS current $J_{i_1 \ldots i_s}$ lies in one supermultiplet and the anomalous term $\mathcal{X}_{i_1 \ldots i_{s-1}}$ lies in a different supermultiplet. In this section we identify examples of such currents and anomalies and the supermultiplets that contain them. The supermultiplets containing the HS currents (with spin higher than 2) are singlets under SU(4) and saturate the $\mathcal{N} = 4$ superconformal unitarity bounds in the free theory. In the free theory, these multiplets assemble together and with the 1/2 BPS ultra-short $\mathcal{N} = 4$ supercurrent multiplet form the doubleton representation of the higher spin group $hs(2, 2|4)$ [7−11].

To fix the notation, let us recall that $\mathcal{N} = 4$ SYM consists of the following elementary fields - all transforming in the adjoint representation of the gauge group - six scalars ϕ_I $I = 1 \ldots 6$ transforming in the vector representation of the SO(6) $\sim$ SU(4) R-symmetry group, four complex fermions $\lambda_{\alpha A}$, $\bar{\lambda}^A_{\dot{\alpha}}$ $A = 1 \ldots 4$ transforming in the spinor representation of SO(6), isomorphic to the fundamental of SU(4), and the gauge field strength F_{ij} which is a singlet. Here $\alpha(\dot{\alpha})$ are left-handed (right-handed) Weyl spinor indices and $i, j = 1 \ldots 4$ are vector indices. We will often represent the field strength tensor as a bispinor $F_{\alpha\beta}$ defined via the four dimensional sigma-matrices as

$$F_{ij} = \frac{1}{2}(\sigma^{\alpha\beta}_{ij} F_{\alpha\beta} + \bar{\sigma}^{\dot{\alpha}\dot{\beta}}_{ij} \bar{F}_{\dot{\alpha}\dot{\beta}})\,. \tag{5.2}$$

We will also convert between SO(6) indices and SU(4) indices using SO(6) Γ-matrices, so that $W_{AB} = \Gamma^I_{AB} W_I$. We use $\mathcal{N} = 4$ (on-shell) Minkowski superspace to describe the supermultiplets. This superspace has coordinates $(x^i, \theta^{\alpha A}, \bar\theta^{\dot\alpha}_A)$ and corresponding super-covariant derivatives $(\partial_i, D_{\alpha A}, \bar D^A_{\dot\alpha})$. These derivatives all (anti)commute with each other except for the following non-trivial anti-commutator

$$\{D_{\alpha A}, \bar D^B_{\dot\alpha}\} = i\delta^B_A \partial_{\alpha\dot\alpha}\,. \tag{5.3}$$

The elementary fields of $\mathcal{N}=4$ SYM occur in the field strength superfield W_I whose bottom $(\theta = \bar\theta = 0)$ component is the scalar ϕ_I. We also use the superfields $\Lambda_{\alpha A}, \bar\Lambda^B_{\dot\alpha}$ whose bottom components are the spinors $\lambda_{\alpha A}, \bar\lambda^B_{\dot\alpha}$. Supermultiplets can be formed by taking particular combinations of gauge invariant products of these 'singleton' superfields acted on by space-time derivatives.

5.1 Free $\mathcal{N}=4$

The superfields we are interested in, which contain HS conserved currents in the absence of interactions, have the form

$$\begin{aligned}
\mathcal{H}^{(s,s)}_{\alpha_1\ldots\alpha_s\dot\alpha_1\ldots\dot\alpha_s} &:= \\
:= \sum_{k=0}^{s}(-1)^k \binom{s}{k}^2 &\mathrm{Tr}(\partial^k_{(\alpha_1\ldots\alpha_k(\dot\alpha_1\ldots\dot\alpha_k} W_I \partial^{s-k}_{\alpha_{k+1}\ldots\alpha_s)\dot\alpha_{k+1}\ldots\dot\alpha_s)} W_I) - \\
-4i\sum_{k=0}^{s}(-1)^k \binom{s}{k}\binom{s}{k-1} &\mathrm{Tr}\Big(\partial^{k-1}_{(\alpha_1\ldots\alpha_{k-1}(\dot\alpha_1\ldots\dot\alpha_{k-1}} \Lambda^A_{\alpha_k} \partial^{s-k}_{\alpha_{k+1}\ldots\alpha_s)\dot\alpha_k\ldots\dot\alpha_{s-1}} \bar\Lambda_{\dot\alpha_k)A}\Big) - \\
-4\sum_{k=0}^{s}(-1)^k \binom{s}{k}\binom{s}{k-2} &\mathrm{Tr}\Big(\partial^{k-2}_{(\alpha_1\ldots\alpha_{k-2}(\dot\alpha_1\ldots\dot\alpha_{k-2}} F_{\alpha_{k-1}\alpha_k} \partial^{s-k}_{\alpha_{k+2}\ldots\alpha_s)\dot\alpha_{k-1}\ldots\dot\alpha_{s-2}} \bar F_{\dot\alpha_{s-1}\dot\alpha_s)}\Big)
\end{aligned} \tag{5.4}$$

with s even.[9] We have converted all Lorentz indices into Weyl spinor indices and the undotted spinor indices and dotted spinor indices are separately symmetrised.

The simplest way of obtaining this expression for the supercurrents is to adapt the expressions found for purely bosonic currents involving only scalar fields given in [7, 9, 34]

$$j^{(s,s)}_{\alpha_1\ldots\alpha_s\dot\alpha_1\ldots\dot\alpha_s} := \sum_{k=0}^{s}(-1)^k \binom{s}{k}^2 \mathrm{Tr}(\partial^k_{(\alpha_1\ldots\alpha_k(\dot\alpha_1\ldots\dot\alpha_k}\phi\, \partial^{s-k}_{\alpha_{k+1}\ldots\alpha_s)\dot\alpha_{k+1}\ldots\dot\alpha_s)}\phi) \tag{5.5}$$

using $\mathcal{N}=4$ analytic superspace [36]. Since this expression is a primary operator in a conformal field theory in four dimensions it can be extended to a superconformal primary operator in $\mathcal{N}=4$ by simply replacing space-time derivatives $\partial_{\alpha\dot\alpha}$ with analytic superspace derivatives $\partial_{\mathcal{A}\mathcal{A}'}$ and the scalar ϕ with the field strength superfield in analytic superspace $\mathcal{W}$ (for notation and a review of superindices in analytic superspace see [37])

$$\begin{aligned}
&\mathcal{H}^{(s,s)}_{\mathcal{A}_1\ldots\mathcal{A}_{s+2}\mathcal{A}'_1\ldots\mathcal{A}'_{s+2}} := \\
&\quad := \sum_{k=0}^{s+2}(-1)^k \binom{s+2}{k}^2 \mathrm{Tr}(\partial^k_{(\mathcal{A}_1\ldots\mathcal{A}_k(\mathcal{A}'_1\ldots\mathcal{A}'_k}\mathcal{W}\, \partial^{s-k+2}_{\mathcal{A}_{k+1}\ldots\mathcal{A}_{s+2})\mathcal{A}'_{k+1}\ldots\mathcal{A}'_{s+2})}\mathcal{W})\,.
\end{aligned} \tag{5.6}$$

[9] For odd s one gets conformal descendants i.e. total derivatives. Indeed when s is odd the first line of the above formula for $\mathcal{H}^{(s,s)}$ vanishes.

JHEP08(2005)088

Here we have split the internal index $A = (a, a')$ so that $a = (1, 2)$ $a' = (3, 4)$ and then the superindices are given as $\mathcal{A} = (\alpha, a)$ $\mathcal{A}' = (\dot\alpha, a')$: where $\alpha, \dot\alpha$ are thought of as even indices and a, a' as odd indices. Symmetrisation of the superindices is generalised meaning that the α indices are symmetrised but the a indices are antisymmetrised. Note that in this expression the number of analytic superspace derivatives is $s + 2$ not s. The case $s = 0$ was first given in an analytic superspace context in [38]. It corresponds to the (in)famous Konishi multiplet $\mathcal{K}$ that requires a separate treatment in the anomaly structure which follows [40]. In fact the above expression is also valid for $s = -2$ which corresponds to the energy-momentum multiplet.

Having found the expression for the supercurrents in analytic superspace we wish to re-express them in the more familiar Minkowski superspace. In order to do this we write down the lowest dimension component of the above operator given by setting two of the unprimed superindices to be internal and the rest to be external and similarly for the primed indices. Ie $\mathcal{A}_i = \alpha_i,\ i = 1 \dots s,\ \mathcal{A}_{s+1} = a_{s+1},\ \mathcal{A}_{s+2} = a_{s+2}$ and similarly for the primed superindices, meaning that we will have s remaining α and $\dot\alpha$ indices as we would expect. Note that we can not have any more internal indices than two since the internal indices are anti-symmetrised and there are only two of them. After taking into account all possible placements of the different types of indices one obtains (5.4).

Other superfields which we are interested in are the short supermultiplets $\mathcal{M}_A^{(s-1,s)}$, $\bar{\mathcal{M}}^{A(s,s-1)}$ and $\mathcal{N}_A^{B(s-1,s-1)}$ which for $s \geq 2$ have the form[10]

$$\mathcal{M}_A := \sum_{\substack{a,b,c\geq 0\\ a+b+c=s-1}} A(a,b,c+1)\, \mathrm{Tr}\left(\partial^a \bar\Lambda_B [\partial^b W_{AC}, \partial^c W^{BC}]\right) + \cdots\,, \tag{5.7}$$

$$\mathcal{N}_A^B := \sum_{\substack{a,b,c,d\geq 0\\ a+b+c+d=s-1}} B(a,b,c,d)\, \mathrm{Tr}\left([\partial^a W_{DF}, \partial^b W^{BF}][\partial^c W_{AC}, \partial^d W^{DC}]\right) + \cdots\,, \tag{5.8}$$

where

$$A(a,b,c) := \frac{4\,i\,c^2}{(a+1)} \frac{(s-1)!\,(s+1)!}{a!\,b!\,c!} (k_a + k_b + k_c) \tag{5.9}$$

$$k_a := \frac{(-1)^a}{a!\,(s+1-a)!} \tag{5.10}$$

$$B(a,b,c,d) := A(a+b,c,d+1) \frac{s+1}{s} \binom{a+b}{a} \frac{b+1}{a+b+2} \tag{5.11}$$

and where we display only the terms involving the combination of fields $\bar\Lambda W^2$ for $\mathcal{M}_A^{(s-1,s)}$ and only terms involving W^4 for $\mathcal{N}_B^{A(s-1,s-1)}$. All spinor indices (here suppressed) are symmetrised. The dots refer to the remaining terms involving other combinations of the fundamental fields and for $\mathcal{N}$ they also contain the SU(4) trace needed to project onto the **15** dimensional irrep. For example $\mathcal{M}_A^{(s-1,s)}$ contains further terms involving $\bar\Lambda^2\Lambda$, $W\Lambda\bar F$

[10] We thank E. Sokatchev for pointing out some missing terms in the definition of $\mathcal{M}_A^{(s-1,s)}$ in the original version of this paper. The multiplets $\mathcal{M}_A^{(s-1,s)}$, $\bar{\mathcal{M}}^{A(s,s-1)}$ and $\mathcal{N}_A^{B(s-1,s-1)}$ have since also been given in [39] and they agree with the expressions given here.

JHEP08(2005)088

and $\bar{\Lambda}F\bar{F}$. The full expressions will be quite complicated and it would be interesting to see if there is a simpler expression for the anomalies in analytic superspace as there was for the supercurrents themselves in (5.6).

The superscript (s,t) corresponds to the number (s) of un-dotted spinor indices and (t) dotted indices which we will largely suppress. These superfields are subject to the following shortening conditions in the free theory

$$D^{\alpha}_{A}\mathcal{H}_{\alpha\ldots}=0 \qquad \bar{D}^{\dot{\alpha}A}\mathcal{H}_{\dot{\alpha}\ldots}=0 \tag{5.12}$$

$$D^{\alpha}_{A}\bar{\mathcal{M}}^{B}_{\alpha\ldots}-\frac{1}{4}\delta^{B}_{A}D^{\alpha}_{C}\bar{\mathcal{M}}^{C}_{\ldots}=0 \qquad \bar{D}^{\dot{\alpha}A}\mathcal{M}_{B\dot{\alpha}\ldots}-\frac{1}{4}\delta^{A}{}_{B}\bar{D}^{\dot{\alpha}C}\mathcal{M}_{C\dot{\alpha}\ldots}=0 \tag{5.13}$$

$$D^{\alpha}_{(A}\mathcal{M}_{B)\alpha\ldots}=0 \qquad \bar{D}^{\dot{\alpha}(A}\bar{\mathcal{M}}^{B)}_{\dot{\alpha}\ldots}=0 \tag{5.14}$$

$$D^{\alpha}_{(C}\mathcal{N}^{A}_{B)\alpha\ldots}-\frac{1}{5}D^{\alpha}_{E}\mathcal{N}^{E}_{\alpha\ldots(B}\delta^{A}_{C)}=0 \qquad \bar{D}^{\dot{\alpha}(C}\mathcal{N}^{B)}_{A\dot{\alpha}\ldots}-\frac{1}{5}\bar{D}^{\dot{\alpha}E}\mathcal{N}^{(B}_{E\dot{\alpha}\ldots}\delta^{C)}_{A}=0\,. \tag{5.15}$$

All of these superfields lie on the superconformal bounds. It can be explicitly shown that the supercurrents of eq. (5.4) satisfy these constraints. Note that from equation (5.12) it is straightforward to deduce that the $\theta=0$ component of $\mathcal{H}^{(s,s)}$ is a spin s conserved current in the free theory:

$$4i\partial^{\alpha\dot{\alpha}}\mathcal{H}^{(s,s)}_{\alpha\dot{\alpha}\ldots}=\left\{D^{\alpha}_{A},\bar{D}^{\dot{\alpha}A}\right\}\mathcal{H}^{(s,s)}_{\alpha\dot{\alpha}\ldots}=0\,. \tag{5.16}$$

For $s=0$ the superfield $\mathcal{H}^{(0,0)}=\mathcal{K}:=\mathrm{Tr}(W_IW_I)$ satisfies a generalized linearity constraint [38, 40]

$$\frac{1}{4}\bar{D}^{A}_{\dot{\alpha}}\bar{D}^{B\dot{\alpha}}\mathcal{K}=0\,, \tag{5.17}$$

that makes it a semishort multiplet and in particular implies that the $\theta\bar{\theta}$ components (a singlet and a **15**) are conserved vector currents. In fact all $\theta^s\bar{\theta}^s$ are conserved spin s currents.

5.2 Interactions and anomalies

In the interacting theory the operators (5.4), (5.7), (5.8) are given by simply replacing the space-time derivative with the covariant derivative (using the conventions given in the appendix of [41]). However the equations (5.12), (5.13) become anomalous. In fact the four superfields combine to make a single long superfield. Equations (5.12), (5.13) become

$$D^{\alpha}_{A}\mathcal{H}_{\alpha\ldots}=g\mathcal{M}_{A\ldots} \qquad \bar{D}^{\dot{\alpha}A}\mathcal{M}_{B\dot{\alpha}\ldots}-\frac{1}{4}\delta^{A}_{B}\bar{D}^{\dot{\alpha}C}\mathcal{M}_{C\dot{\alpha}\ldots}=g\mathcal{N}^{A}_{B} \tag{5.18}$$

$$\bar{D}^{\dot{\alpha}A}\mathcal{H}_{\dot{\alpha}\ldots}=g\bar{\mathcal{M}}^{A}_{\ldots} \qquad D^{\alpha}_{A}\bar{\mathcal{M}}^{B}_{\alpha\ldots}-\frac{1}{4}\delta^{B}_{A}D^{\alpha}_{C}\bar{\mathcal{M}}^{C}_{\alpha\ldots}=g\mathcal{N}^{B}_{A}\,. \tag{5.19}$$

We see that $\mathcal{H}$ 'swallows up' $\mathcal{M},\bar{\mathcal{M}}$ and $\mathcal{N}$ to form a single superfield which is now unconstrained and hence corresponds to a long supermultiplet. Indeed we have used these equations to calculate $\mathcal{M}$ and $\mathcal{N}$ from $\mathcal{H}$ in the first place. Note that equations (5.14), (5.15) still hold: they are now automatically satisfied. For example (5.14) becomes

$$D^{\alpha}_{(A}M_{B)\alpha\ldots}=\frac{1}{g}D^{\alpha}_{(A}D^{\beta}_{B)}\mathcal{H}_{\alpha\beta\ldots}=0\,, \tag{5.20}$$

which is identically satisfied since $\mathcal{H}^{(s,s)}$ is totally symmetric in its spinor indices (of a given chirality) and the covariant derivatives anti-commute: $\{D_{\alpha A}, D_{\beta B}\} = 0$.

We are now in a position to see which multiplets the anomaly sits in. The conservation condition (5.16) becomes

$$4i\partial^{\alpha\dot{\alpha}}\mathcal{H}^{(s,s)}_{\alpha\dot{\alpha}\ldots} = \left\{D^{\alpha}_{A}, \bar{D}^{\dot{\alpha}A}\right\}\mathcal{H}^{(s,s)}_{\alpha\dot{\alpha}\ldots} = g(D^{\alpha}_{A}\bar{\mathcal{M}}^{A}_{\alpha\ldots} + \bar{D}^{\dot{\alpha}A}\mathcal{M}_{A\dot{\alpha}\ldots}) \tag{5.21}$$

and so we see that the anomaly $\mathcal{X}_{i_1\ldots i_{s-1}}$ of (5.1) is the real part of $D^{\alpha}_{A}\bar{\mathcal{M}}^{A}_{\alpha\ldots}$.

Note that the anomaly $\mathcal{X}^{(s-1,s-1)}$, though totally symmetric in its $s-1$ vector indices (or as many undotted and dotted spinor indices) is not a current itself even in the absence of interactions. This can be deduced from conformal symmetry since it does not saturate the conformal unitarity bounds of the current type. Indeed $\mathcal{H}^{(s,s)}$ has spin s and dilation weight $s+2$ and hence saturates the relevant unitary bound. Whereas the anomaly $\mathcal{X}^{(s-1,s-1)}$ has spin $s-1$ but dilation weight $s+3 = (s-1)+2+2$ and hence does not.

For $s = 0$ the $\mathcal{N} = 4$ Konishi anomaly [40, 42] reads

$$\frac{1}{4}\bar{D}^{A}_{\dot{\alpha}}\bar{D}^{B\dot{\alpha}}\mathcal{K} = g\mathcal{E}^{AB}\,, \tag{5.22}$$

where in turn the superfield

$$\mathcal{E}^{AB} = \frac{1}{3!}\Gamma^{AB}_{IJK}\,\mathrm{Tr}(W^{I}[W^{J}, W^{K}]) \tag{5.23}$$

is 1/8 BPS in the free theory and satisfies

$$\frac{1}{4}D^{\alpha}_{A}D_{B\alpha}\mathcal{E}^{CD} = g\mathcal{V}^{CD}_{AB}\,, \tag{5.24}$$

where finally the superfield

$$\mathcal{V}^{CD}_{AB} = \frac{1}{8}\Gamma^{(C}_{IJ(A}\Gamma^{D)}_{KLB)}\,\mathrm{Tr}([W^{I}, W^{J}][W^{K}, W^{L}]) - \cdots \tag{5.25}$$

is a 1/4 BPS multiplet in free theory [43]. Terms in dots denote subtraction of SU(4) traces needed to project $\mathcal{V}^{CD}_{AB}$ onto the **84** dimensional irrep with Dynkin labels $[2, 0, 2]$.

More general shortening conditions that involve the boundary counterparts of the KK excitations of the HS gauge fields can be consider as in [44, 10] but we will not dwell on such cases.

6. Conclusions and perspectives

To conclude we would like to present a synthesis of our results and attempt to put them in the perspective of *La Grande Bouffe*. In agreement with previous analyses [20, 21], we have found that, contrary to flat spacetime but consistently with holography, the relevant Stückelberg field for the spontaneous breaking of a HS gauge symmetry associated to an originally 'massless' spin s field in the AdS bulk is a genuinely massive spin $s-1$ field. The case $s = 1$ is an exception to the rule while already $s = 2$ follows the pattern. The general strategy was to covariantise the flat space field equations, but it is also possible to apply the same technique directly to the lagrangian [20, 21], whose precise form is fixed by requiring that it propagates the right degrees of freedom. In flat space, the procedure was to obtain the D-dimensional equations via dimensional reduction. It would be interesting to see whether such a derivation is possible for our equations directly in AdS, along the lines of ref. [45].

We have restricted our attention to the case of totally symmetric bosonic tensors, although for AdS_5 more general possibilities are allowed. Massive fields of mixed symmetry in AdS_5 were discussed in [26], while flat space equations for gauge fields in arbitrary representations of the Lorentz group were introduced in [46] and more recently discussed in [47, 48]. It would be interesting to extend our results to these cases, possibly by means of BRS dimensional reduction [49]. The manifestly supersymmetric analysis of section 5 shows that at least for the bulk theory dual to $\mathcal{N} = 4$ it should be possible to relate the Higgs mechanism for mixed symmetry tensors to the one described in the previous sections. Moreover, very much as all 'conserved' HS current supermultiplets $\mathcal{H}^{(s,s)}$ become part of a unique $HS(2,2|4)$ doubleton multiplet, the anomalous terms $\mathcal{M}_A^{(s,s-1)}$ (and its conjugate $\bar{\mathcal{M}}^{A(s-1,s)}$) and $\mathcal{N}_B^{A(s-1,s-1)}$ become part of the totally antisymmetric tripleton and 'window' tetrapleton respectively [11]. Free massless field equations for the doubleton have been derived by Sezgin and Sundell [7, 8] starting from Vasiliev equations [17–19]. The latter in turn require the introduction of a master gauge connection Ω, comprising all gauge fields with $s \geq 2$, and a master scalar field Φ, comprising the low spin ($s = 0$ and $s = 1/2$) non-gauge fields as well as the generalized antisymmetric tensors satisfying a 'massive' self-duality constraint. We thus expect it to be possible to introduce 'massive' master fields for the relevant tripleton and tetrapleton and couple them to the doubleton in a HS symmetric fashion in AdS very much as we have done in components. We hope to soon report on this.

Finally it is interesting to observe that the discontinuity of the massive equations in flat space is reminiscent of the van Dam-Veltman-Zakharov discontinuity of massive gravity [50]. In [51] it was shown that no discontinuity occurs in AdS, and the fact that massive gravity has a different behaviour in AdS than in flat space in the massless limit could have some connection with the results presented in this paper and with the transition studied in [52].

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References

[1] J.M. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [`hep-th/9711200`];
S. Ferrara and C. Fronsdal, *Conformal Maxwell theory as a singleton field theory on AdS_5, IIB three branes and duality*, *Class. and Quant. Grav.* **15** (1998) 2153 [`hep-th/9712239`];
Gauge fields as composite boundary excitations, *Phys. Lett.* **B 433** (1998) 19 [`hep-th/9802126`];
E. Witten, *Anti-de Sitter space and holography*, `hep-th/9802150`;
S. Ferrara and C. Fronsdal, *Gauge fields as composite boundary excitations*, *Phys. Lett.* **B 433** (1998) 19 [`hep-th/9802126`];
S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from non-critical string theory*, *Phys. Lett.* **B 428** (1998) 105 [`hep-th/9802109`].

[2] For review see e.g. E. D'Hoker and D.Z. Freedman, *Supersymmetric gauge theories and the AdS/CFT correspondence*, `hep-th/0201253`;
O. Aharony, S.S. Gubser, J.M. Maldacena, H. Ooguri and Y. Oz, *Large-N field theories, string theory and gravity*, *Phys. Rept.* **323** (2000) 183 [`hep-th/9905111`];
M. Bianchi, *(non-)perturbative tests of the AdS/CFT correspondence*, *Nucl. Phys.* **102** *(Proc. Suppl.)* (2001) 56 [`hep-th/0103112`].

[3] D. Berenstein, J.M. Maldacena and H. Nastase, *Strings in flat space and pp waves from $N = 4$ super Yang-Mills*, *JHEP* **04** (2002) 013 [`hep-th/0202021`];
N.R. Constable, D.Z. Freedman, M. Headrick and S. Minwalla, *Operator mixing and the BMN correspondence*, *JHEP* **10** (2002) 068 [`hep-th/0209002`];
N. Beisert, C. Kristjansen, J. Plefka, G.W. Semenoff and M. Staudacher, *Bmn correlators and operator mixing in $N = 4$ super Yang-Mills theory*, *Nucl. Phys.* **B 650** (2003) 125 [`hep-th/0208178`].

[4] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *A semi-classical limit of the gauge/string correspondence*, *Nucl. Phys.* **B 636** (2002) 99 [`hep-th/0204051`];
A.A. Tseytlin, *Semiclassical strings and AdS/CFT*, `hep-th/0409296`;
J. Engquist, J.A. Minahan and K. Zarembo, *Yang-Mills duals for semiclassical strings on $AdS_5 \times S^5$*, *JHEP* **11** (2003) 063 [`hep-th/0310188`];
N. Beisert, J.A. Minahan, M. Staudacher and K. Zarembo, *Stringing spins and spinning strings*, *JHEP* **09** (2003) 010 [`hep-th/0306139`].

[5] B. Sundborg, *The hagedorn transition, deconfinement and $N = 4$ SYM theory*, *Nucl. Phys.* **B 573** (2000) 349 [`hep-th/9908001`];
P. Haggi-Mani and B. Sundborg, *Free large-N supersymmetric Yang-Mills theory as a string theory*, *JHEP* **04** (2000) 031 [`hep-th/0002189`];
B. Sundborg, *Stringy gravity, interacting tensionless strings and massless higher spins*, *Nucl. Phys.* **102** *(Proc. Suppl.)* (2001) 113 [`hep-th/0103247`]. E. Witten, *Spacetime reconstruction*, talk at *JHS 60 conference*, Caltech, 3-4 Nov 2001
`http://quark.caltech.edu/jhs60/witten/1.html`;
A. M. Polyakov, *Gauge fields and space-time*, *Int. J. Mod. Phys.* **A 17S1** (2002) 119 [`hep-th/0110196`].

[6] I.R. Klebanov and A.M. Polyakov, *AdS dual of the critical O(N) vector model*, *Phys. Lett.* **B 550** (2002) 213 [`hep-th/0210114`];
L. Girardello, M. Porrati and A. Zaffaroni, *3-D interacting cfts and generalized Higgs phenomenon in higher spin theories on AdS*, *Phys. Lett.* **B 561** (2003) 289 [`hep-th/0212181`];

E. Sezgin and P. Sundell, *Holography in 4D (super) higher spin theories and a test via cubic scalar couplings*, *JHEP* **07** (2005) 044 [hep-th/0305040];
R.G. Leigh and A.C. Petkou, $\mathrm{Sl}(2,\mathbb{Z})$ *action on three-dimensional cfts and holography*, *JHEP* **12** (2003) 020 [hep-th/0309177];
R.G. Leigh and A.C. Petkou, *Holography of the $N=1$ higher-spin theory on AdS_4*, *JHEP* **06** (2003) 011 [hep-th/0304217].

[7] E. Sezgin and P. Sundell, *Massless higher spins and holography*, *Nucl. Phys.* **B 644** (2002) 303 [hep-th/0205131], erratum *ibid.* **B660** (2003) 403.

[8] E. Sezgin and P. Sundell, *Towards massless higher spin extension of $D=5$, $N=8$ gauged supergravity*, *JHEP* **09** (2001) 025 [hep-th/0107186];
E. Sezgin and P. Sundell, *Doubletons and 5D higher spin gauge theory*, *JHEP* **09** (2001) 036 [hep-th/0105001].

[9] S.E. Konstein, M.A. Vasiliev and V.N. Zaikin, *Conformal higher spin currents in any dimension and AdS/CFT correspondence*, *JHEP* **12** (2000) 018 [hep-th/0010239].

[10] N. Beisert, M. Bianchi, J.F. Morales and H. Samtleben, *On the spectrum of AdS/CFT beyond supergravity*, *JHEP* **02** (2004) 001 [hep-th/0310292].

[11] N. Beisert, M. Bianchi, J.F. Morales and H. Samtleben, *Higher spin symmetry and $N=4$ SYM*, *JHEP* **07** (2004) 058 [hep-th/0405057].

[12] M. Bianchi, J.F. Morales and H. Samtleben, *On stringy $AdS_5 \times S^5$ and higher spin holography*, *JHEP* **07** (2003) 062 [hep-th/0305052].

[13] M. Bianchi, B. Eden, G. Rossi and Y.S. Stanev, *On operator mixing in $N=4$ SYM*, *Nucl. Phys.* **B 646** (2002) 69 [hep-th/0205321];
B. Eden and E. Sokatchev, *On the ope of 1/2 BPS short operators in $N=4$ SCFT(4)*, *Nucl. Phys.* **B 618** (2001) 259 [hep-th/0106249];
G. Arutyunov, B. Eden and E. Sokatchev, *On non-renormalization and ope in superconformal field theories*, *Nucl. Phys.* **B 619** (2001) 359 [hep-th/0105254];
F.A. Dolan and H. Osborn, *Superconformal symmetry, correlation functions and the operator product expansion*, *Nucl. Phys.* **B 629** (2002) 3 [hep-th/0112251];
M. Bianchi, G. Rossi and Y.S. Stanev, *Surprises from the resolution of operator mixing in $N=4$ SYM*, *Nucl. Phys.* **B 685** (2004) 65 [hep-th/0312228];
P.J. Heslop and P.S. Howe, *Opes and 3-point correlators of protected operators in $N=4$ SYM*, *Nucl. Phys.* **B 626** (2002) 265 [hep-th/0107212].

[14] N. Beisert, C. Kristjansen and M. Staudacher, *The dilatation operator of $N=4$ super Yang-Mills theory*, *Nucl. Phys.* **B 664** (2003) 131 [hep-th/0303060];
N. Beisert, *The dilatation operator of $N=4$ super Yang-Mills theory and integrability*, *Phys. Rept.* **405** (2005) 1 [hep-th/0407277] *The complete one-loop dilatation operator of $N=4$ super Yang-Mills theory*, *Nucl. Phys.* **B 676** (2004) 3 [hep-th/0307015].

[15] J.A. Minahan and K. Zarembo, *The bethe-ansatz for $N=4$ super Yang-Mills*, *JHEP* **03** (2003) 013 [hep-th/0212208];
N. Beisert and M. Staudacher, *The $N=4$ SYM integrable super spin chain*, *Nucl. Phys.* **B 670** (2003) 439 [hep-th/0307042];
N. Beisert, V. Dippel and M. Staudacher, *A novel long range spin chain and planar $N=4$ super Yang-Mills*, *JHEP* **07** (2004) 075 [hep-th/0405001].

[16] N. Beisert, S. Frolov, M. Staudacher and A.A. Tseytlin, *Precision spectroscopy of AdS/CFT*, *JHEP* **10** (2003) 037 [hep-th/0308117];
B. Eden, C. Jarczak and E. Sokatchev, *A three-loop test of the dilatation operator in $N = 4$ SYM*, *Nucl. Phys.* **B 712** (2005) 157 [hep-th/0409009];
G. Arutyunov, S. Frolov, J. Russo and A.A. Tseytlin, *Spinning strings in $AdS_5 \times S^5$ and integrable systems*, *Nucl. Phys.* **B 671** (2003) 3 [hep-th/0307191].

[17] M.A. Vasiliev, *Actions, charges and off-shell fields in the unfolded dynamics approach*, hep-th/0504090;
K.B. Alkalaev, O.V. Shaynkman and M.A. Vasiliev, *Lagrangian formulation for free mixed-symmetry bosonic gauge fields in Ads_d*, hep-th/0501108;
A.S. Matveev and M.A. Vasiliev, *On dual formulation for higher spin gauge fields in Ads_d*, *Phys. Lett.* **B 609** (2005) 157 [hep-th/0410249];
M.A. Vasiliev, *Higher spin superalgebras in any dimension and their representations*, *JHEP* **12** (2004) 046 [hep-th/0404124];
K.B. Alkalaev, O.V. Shaynkman and M.A. Vasiliev, *On the frame-like formulation of mixed-symmetry massless fields in Ads_d*, *Nucl. Phys.* **B 692** (2004) 363 [hep-th/0311164];
M.A. Vasiliev, *Nonlinear equations for symmetric massless higher spin fields in Ads_d*, *Phys. Lett.* **B 567** (2003) 139 [hep-th/0304049];
V.E. Didenko and M.A. Vasiliev, *Free field dynamics in the generalized AdS (super)space*, *J. Math. Phys.* **45** (2004) 197 [hep-th/0301054];
K.B. Alkalaev and M.A. Vasiliev, *$N = 1$ supersymmetric theory of higher spin gauge fields in AdS_5 at the cubic level*, *Nucl. Phys.* **B 655** (2003) 57 [hep-th/0206068].

[18] A. Sagnotti, E. Sezgin and P. Sundell, *On higher spins with a strong* $\mathrm{Sp}(2,\mathbb{R})$ *condition*, hep-th/0501156;
A. Sagnotti and M. Tsulaia, *On higher spins and the tensionless limit of string theory*, *Nucl. Phys.* **B 682** (2004) 83 [hep-th/0311257];
D. Francia and A. Sagnotti, *On the geometry of higher-spin gauge fields*, *Class. Quant. Grav.* **20** (2003) S473–S486 [hep-th/0212185];
D. Francia and A. Sagnotti, *Free geometric equations for higher spins*, *Phys. Lett.* **B 543** (2002) 303 [hep-th/0207002].

[19] For review see e.g. M.A. Vasiliev, *Progress in higher spin gauge theories*, hep-th/0104246; *Higher spin gauge theory*, *Usp. Fiz. Nauk.* **173** (2003) 226; *Higher spin gauge theories in any dimension*, *Comptes Rendus Physique* **5** (2004) 1101–1109 [hep-th/0409260]; *Higher spin gauge theories in various dimensions*, *Fortschr. Phys.* **52** (2004) 702 [hep-th/0401177];
N. Bouatta, G. Compere and A. Sagnotti, *An introduction to free higher-spin fields*, hep-th/0409068;
X. Bekaert, S. Cnockaert, C. Iazeolla and M.A. Vasiliev, *Nonlinear higher spin theories in various dimensions*, hep-th/0503128;
M. Bianchi and V. Didenko, *'Massive' higher spin multiplets and holography*, hep-th/0502220.

[20] S. Deser and A. Waldron, *Gauge invariances and phases of massive higher spins in AdS*, *Phys. Rev. Lett.* **87** (2001) 031601 [hep-th/0102166]; *Partial masslessness of higher spins in AdS*, *Nucl. Phys.* **B 607** (2001) 577 [hep-th/0103198]; *Null propagation of partially massless higher spins in AdS and cosmological constant speculations*, *Phys. Lett.* **B 513** (2001) 137 [hep-th/0105181]; *Arbitrary spin representations in de Sitter from dS/CFT with applications to dS supergravity*, *Nucl. Phys.* **B 662** (2003) 379 [hep-th/0301068].

[21] Y.M. Zinoviev, *On massive high spin particles in AdS*, hep-th/0108192.

[22] S.D. Rindani, D. Sahdev and M. Sivakumar, *Dimensional reduction of symmetric higher spin actions, 1. Bosons*, *Mod. Phys. Lett.* **A 4** (1989) 265;
F. Riccioni, *Alcune proprietà dei campi di gauge di spin arbitrario*, Laurea Thesis (in italian), available on: http://people.roma2.infn.it/~stringhe/activities.htm.

[23] M. Bianchi, *Higher spins and stringy* $AdS_5 \times S^5$, *Fortschr. Phys.* **53** (2005) 665 [hep-th/0409304].

[24] M. Bianchi, *Higher spin symmetry (breaking) in* $N = 4$ *SYM and holography*, *Comptes Rendus Physique* **5** (2004) 1091–1099 [hep-th/0409292].

[25] M. Porrati, *Massive gravity in AdS and Minkowski backgrounds*, hep-th/0409172; *Higgs phenomenon for the graviton in AdS space*, *Mod. Phys. Lett.* **A 18** (2003) 1793 [hep-th/0306253]; *Higgs phenomenon for 4-d gravity in anti de Sitter space*, *JHEP* **04** (2002) 058 [hep-th/0112166].

[26] L. Brink, R.R. Metsaev and M.A. Vasiliev, *How massless are massless fields in AdS(d)*, *Nucl. Phys.* **B 586** (2000) 183 [hep-th/0005136];
R.R. Metsaev, *Massless arbitrary spin fields in AdS(5)*, *Phys. Lett.* **B 531** (2002) 152 [hep-th/0201226]; *Light-cone form of field dynamics in AdS space-time*, *Int. J. Mod. Phys.* **A16S1C** (2001) 994–997 [hep-th/0011112].

[27] R.R. Metsaev, *Mixed symmetry massive fields in* AdS_5, *Class. and Quant. Grav.* **22** (2005) 2777 [hep-th/0412311];
R.R. Metsaev, *Massive totally symmetric fields in AdS(d)*, *Phys. Lett.* **B 590** (2004) 95 [hep-th/0312297].

[28] M. Henningson and K. Skenderis, *Holography and the Weyl anomaly*, *Fortschr. Phys.* **48** (2000) 125 [hep-th/9812032];
M. Henningson and K. Skenderis, *The holographic Weyl anomaly*, *JHEP* **07** (1998) 023 [hep-th/9806087];
L. Girardello, M. Petrini, M. Porrati and A. Zaffaroni, *Novel local CFT and exact results on perturbations of* $N = 4$ *super Yang-Mills from AdS dynamics*, *JHEP* **12** (1998) 022 [hep-th/9810126];
O. Aharony, J. Pawelczyk, S. Theisen and S. Yankielowicz, *A note on anomalies in the AdS/CFT correspondence*, *Phys. Rev.* **D 60** (1999) 066001 [hep-th/9901134];
M. Bianchi, O. DeWolfe, D.Z. Freedman and K. Pilch, *Anatomy of two holographic renormalization group flows*, *JHEP* **01** (2001) 021 [hep-th/0009156];
M. Bianchi, D.Z. Freedman and K. Skenderis, *Holographic renormalization*, *Nucl. Phys.* **B 631** (2002) 159 [hep-th/0112119];
M. Bianchi, D.Z. Freedman and K. Skenderis, *How to go with an RG flow*, *JHEP* **08** (2001) 041 [hep-th/0105276].

[29] R.G. Leigh and M.J. Strassler, *Exactly marginal operators and duality in four-dimensional* $N = 1$ *supersymmetric gauge theory*, *Nucl. Phys.* **B 447** (1995) 95 [hep-th/9503121].

[30] G. Arutyunov, B. Eden, A.C. Petkou and E. Sokatchev, *Exceptional non-renormalization properties and ope analysis of chiral four-point functions in* $N = 4$ *SYM(4)*, *Nucl. Phys.* **B 620** (2002) 380 [hep-th/0103230];
B. Eden, A.C. Petkou, C. Schubert and E. Sokatchev, *Partial non-renormalisation of the stress-tensor four-point function in* $N = 4$ *SYM and AdS/CFT*, *Nucl. Phys.* **B 607** (2001) 191 [hep-th/0009106];

JHEP08(2005)088

O. Aharony and S.S. Razamat, *Exactly marginal deformations of $N = 4$ SYM and of its supersymmetric orbifold descendants*, *JHEP* **05** (2002) 029 [hep-th/0204045];
O. Aharony, B. Kol and S. Yankielowicz, *On exactly marginal deformations of $N = 4$ SYM and type-IIB supergravity on $AdS_5 \times S^5$*, *JHEP* **06** (2002) 039 [hep-th/0205090].

[31] O. Aharony, Y.E. Antebi, M. Berkooz and R. Fishman, *'holey sheets': pfaffians and subdeterminants as D-brane operators in large-N gauge theories*, *JHEP* **12** (2002) 069 [hep-th/0211152];
O. Aharony, M. Berkooz and E. Silverstein, *Multiple-trace operators and non-local string theories*, *JHEP* **08** (2001) 006 [hep-th/0105309].

[32] C. Fronsdal, *Massless fields with integer spin*, *Phys. Rev.* **D 18** (1978) 3624;
J. Fang and C. Fronsdal, *Massless fields with half integral spin*, *Phys. Rev.* **D 18** (1978) 3630.

[33] L.P.S. Singh and C.R. Hagen, *Lagrangian formulation for arbitrary spin, 1. The boson case*, *Phys. Rev.* **D 9** (1974) 898; *Lagrangian formulation for arbitrary spin, 2. The fermion case*, *Phys. Rev.* **D 9** (1974) 910.

[34] A. Mikhailov, *Notes on higher spin symmetries*, hep-th/0201019.

[35] S. Ferrara and A. Zaffaroni, *Bulk gauge fields in AdS supergravity and supersingletons*, hep-th/9807090;
S. Ferrara and E. Sokatchev, *Superconformal interpretation of BPS states in AdS geometries*, *Int. J. Theor. Phys.* **40** (2001) 935 [hep-th/0005151];
S. Ferrara and A. Zaffaroni, *Superconformal field theories, multiplet shortening and the $AdS_5/sCFT(4)$ correspondence*, hep-th/9908163.

[36] G.G. Hartwell and P.S. Howe, *(N, p, q) harmonic superspace*, *Int. J. Mod. Phys.* **A 10** (1995) 3901 [hep-th/9412147];
P.S. Howe and G.G. Hartwell, *A superspace survey*, *Class. and Quant. Grav.* **12** (1995) 1823.

[37] P. Heslop and P.S. Howe, *Harmonic superspaces and superconformal fields*, hep-th/0009217;
P.J. Heslop, *Superfield representations of superconformal groups*, *Class. and Quant. Grav.* **19** (2002) 303 [hep-th/0108235].

[38] P.S. Howe and P.C. West, *AdS/sCFT in superspace*, *Class. and Quant. Grav.* **18** (2001) 3143 [hep-th/0105218].

[39] J. Henn, C. Jarczak and E. Sokatchev, *On twist-two operators in $N = 4$ SYM*, hep-th/0507241.

[40] L. Andrianopoli and S. Ferrara, *Short and long* $\mathrm{SU}(2,2/4)$ *multiplets in the AdS/CFT correspondence*, *Lett. Math. Phys.* **48** (1999) 145 [hep-th/9812067];
L. Andrianopoli and S. Ferrara, **Nonchiral* primary superfields in the $AdS(d+1)/CFT(d)$ correspondence*, *Lett. Math. Phys.* **46** (1998) 265 [hep-th/9807150];
M. Bianchi, S. Kovacs, G. Rossi and Y.S. Stanev, *Anomalous dimensions in $N = 4$ SYM theory at order G^4*, *Nucl. Phys.* **B 584** (2000) 216 [hep-th/0003203];
M. Bianchi, S. Kovacs, G. Rossi and Y.S. Stanev, *On the logarithmic behavior in $N = 4$ SYM theory*, *JHEP* **08** (1999) 020 [hep-th/9906188].

[41] J.M. Drummond, P.J. Heslop, P.S. Howe and S.F. Kerstan, *Integral invariants in $N = 4$ SYM and the effective action for coincident D-branes*, *JHEP* **08** (2003) 016 [hep-th/0305202].

[42] M. Bianchi, S. Kovacs, G. Rossi and Y.S. Stanev, *Properties of the Konishi multiplet in $N = 4$ SYM theory*, *JHEP* **05** (2001) 042 [hep-th/0104016];
B. Eden, C. Jarczak, E. Sokatchev and Y.S. Stanev, *Operator mixing in $N = 4$ SYM: the konishi anomaly revisited*, *Nucl. Phys.* **B 722** (2005) 119 [hep-th/0501077].

[43] A.V. Ryzhov, *Quarter BPS operators in $N = 4$ SYM*, *JHEP* **11** (2001) 046 [hep-th/0109064];
E. D'Hoker, P. Heslop, P. Howe and A.V. Ryzhov, *Systematics of quarter BPS operators in $N = 4$ SYM*, *JHEP* **04** (2003) 038 [hep-th/0301104];
P.J. Heslop and P.S. Howe, *A note on composite operators in $N = 4$ SYM*, *Phys. Lett.* **B 516** (2001) 367 [hep-th/0106238].

[44] V.K. Dobrev and V.B. Petkova, *On the group theoretical approach to extended conformal supersymmetry: classification of multiplets*, *Lett. Math. Phys.* **9** (1985) 287; *On the group-theoretical approach to extended conformal supersymmetry: function space realizations and invariant differential operators*, *Fortschr. Phys.* **35** (1987) 537; *All positive energy unitary irreducible representations of extended conformal supersymmetry*, *Phys. Lett.* **B 162** (1985) 127; F.A. Dolan and H. Osborn, *On short and semi-short representations for four dimensional superconformal symmetry*, *Ann. Phys. (NY)* **307** (2003) 41 [hep-th/0209056]; P.J. Heslop and P.S. Howe, *Aspects of $N = 4$ SYM*, *JHEP* **01** (2004) 058 [hep-th/0307210].

[45] T. Biswas and W. Siegel, *Radial dimensional reduction: (anti) de Sitter theories from flat*, *JHEP* **07** (2002) 005 [hep-th/0203115].

[46] W. Siegel and B. Zwiebach, *Gauge string fields from the light cone*, *Nucl. Phys.* **B 282** (1987) 125;
W. Siegel, *Fields*, hep-th/9912205.

[47] X. Bekaert and N. Boulanger, *Tensor gauge fields in arbitrary representations of GL(D,R): duality and Poincaré lemma*, *Commun. Math. Phys.* **245** (2004) 27 [hep-th/0208058].

[48] P. de Medeiros and C. Hull, *Geometric second order field equations for general tensor gauge fields*, *JHEP* **05** (2003) 019 [hep-th/0303036].

[49] X. Bekaert, I.L. Buchbinder, A. Pashnev and M. Tsulaia, *On higher spin theory: strings, BRST, dimensional reductions*, *Class. Quant. Grav.* **21** (2004) S1457–1464 [hep-th/0312252].

[50] H. van Dam and M.J.G. Veltman, *Massive and massless Yang-Mills and gravitational fields*, *Nucl. Phys.* **B 22** (1970) 397;
V.I. Zakharov, *JETP Lett.* **12** (1970) 312.

[51] M. Porrati, *No van Dam-Veltman-Zakharov discontinuity in AdS space*, *Phys. Lett.* **B 498** (2001) 92 [hep-th/0011152];
P.A. Grassi and P. van Nieuwenhuizen, *No van Dam-Veltman-Zakharov discontinuity for supergravity in AdS space*, *Phys. Lett.* **B 499** (2001) 174 [hep-th/0011278];
S. Deser and A. Waldron, *(dis)continuities of massless limits in spin 3/2-mediated interactions and cosmological supergravity*, *Phys. Lett.* **B 501** (2001) 134 [hep-th/0012014].
A. Karch, E. Katz and L. Randall, *Absence of a VVDZ discontinuity in AdS(AdS)*, *JHEP* **12** (2001) 016 [hep-th/0106261].

[52] O. Aharony, J. Marsano, S. Minwalla, K. Papadodimas and M. Van Raamsdonk, *A first order deconfinement transition in large-N Yang-Mills theory on a small S^3*, *Phys. Rev.* **D 71** (2005) 125018 [hep-th/0502149]; *The hagedorn/deconfinement phase transition in weakly coupled large-N gauge theories*, *Adv. Theor. Math. Phys.* **8** (2004) 603 [hep-th/0310285].